P<X>:=PolynomialRing(Rationals());

R<h,x>:=PolynomialRing(Rationals(),2);

S<h,x>:=FunctionField(Rationals(),2);

PS<t>:=PolynomialRing(S);

print "Z2xZ36 Case";

j13:=(h^2 + 5\*h + 13)\*(h^4+7\*h^3+20\*h^2+19\*h+1)^3\*h^(-1);

//j-invariant for 13-isogeny

E:=EllipticCurve([1,0,0,-36/(j13-1728),-1/(j13-1728)]);

f:=Discriminant(E);

numfact:=Factorization(Numerator(f));

denfact:=Factorization(Denominator(f));

g:=1;

for i:=1 to #numfact do

 if IsOdd(numfact[i][2]) then

 g:=g\*numfact[i][1];

 end if;

end for;

for i:=1 to #denfact do

 if IsOdd(denfact[i][2]) then

 g:=g\*denfact[i][1];

 end if;

end for;

g;

// Discriminant = h^3 + 6\*h^2 + 13\*h

EE:=EllipticCurve([0,6,0,13,0]);

MordellWeilGroup(EE);

//We find the only points on EE are (0:0:1) and (0:1:0)

print "Z2xZ50 Case";

j25:=(h^(10)+10\*h^8+35\*h^6-12\*h^5+50\*h^4-60\*h^3+25\*h^2-60\*h+16)^3\*(h^5+5\*h^3+5\*h-11)^(-1);

//j-invariant for 25-isogeny

E:=EllipticCurve([1,0,0,-36/(j25-1728),-1/(j25-1728)]);

f:=Discriminant(E);

numfact:=Factorization(Numerator(f));

denfact:=Factorization(Denominator(f));

g:=1;

for i:=1 to #numfact do

 if IsOdd(numfact[i][2]) then

 g:=g\*numfact[i][1];

 end if;

end for;

for i:=1 to #denfact do

 if IsOdd(denfact[i][2]) then

 g:=g\*denfact[i][1];

 end if;

end for;

g;

//Discriminant = h^7 + 9\*h^5 + 25\*h^3 - 11\*h^2 + 20\*h - 44

//Now we make the curve y^2 = Discriminant

C:=HyperellipticCurve(X^7+9\*X^5+25\*X^3-11\*X^2+20\*X-44);

AutC:=Automorphisms(C);

G:=AutomorphismGroup(C,[AutC[3]]);

Cg, phi :=CurveQuotient(G);

MordellWeilGroup(Cg);

Points:=PointSearch(Cg,1000);

for P in Points do

 preimageofP:= P @@ phi;

 RationalPoints(preimageofP);

end for;

//We find the only points on C are (1:0:0) and (1:0:1)

P1:=C![1,0,0];

P2:=C![1,0,1];

//Now, we map those to the projective model of C.

PP<X,Y,Z>:=ProjectiveSpace(Rationals(),2);

CC:=Curve(PP,-Y^2\*Z^5+X^7+9\*X^5\*Z^2+25\*X^3\*Z^4-11\*X^2\*Z^5+20\*X\*Z^6-44\*Z^7); C;

yesno, CtoCC:=IsIsomorphic(C,CC);

CtoCC(P1);

CtoCC(P2);