Math 2110Q - Multivariable Calculus
Name:

Section 16.3 Worksheet

1. Complete the sentences:

   (a) A vector field $\mathbf{F}$ is called conservative if

   there exists f such that $\nabla f = F$

   (b) We can check if a vector field is conservative by checking independence of

   paths

   (c) We can check if a vector field is conservative by using partial derivatives to see if

   $$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

2. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

   (a) $\mathbf{F}(x, y) = (y^2 - 2x)\mathbf{i} + 2xy\mathbf{j}$.

   $P = y^2 - 2x$ $\quad \frac{\partial P}{\partial y} = 2y$ ✓

   $Q = 2xy$ $\quad \frac{\partial Q}{\partial x} = 2y$

   both P and Q are continuous on $\mathbb{R}^2$ (no holes).

   $\int P\,dx = xy^2 - x^2 + g(y)$

   $\int Q\,dy = xy^2 + h(x)$

   $\to f = xy^2 - x^2$

   works if $g(y) = 0$

   $h(x) = -x^2$

   (b) $\mathbf{F}(x, y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$.

   $P = ye^x + \sin y$ $\quad \frac{\partial P}{\partial y} = e^x + \cos y$ ✓

   $Q = e^x + x\cos y$ $\quad \frac{\partial Q}{\partial x} = e^x + \cos y$

   $\int P\,dx = ye^x + \sin y\, x + g(y)$

   $\int Q\,dy = ye^x + x\sin y + h(x)$

   both P and Q are continuous on $\mathbb{R}^2$

   works if $g(y) = h(x) = 0$,

   so $f = ye^x + x\sin y$.

3. Show that the line integral

$$\int_C \sin y\,dx + (x\cos y - \sin y)\,dy$$

is independent of path and evaluate the integral where C is any path from $(1,0)$ to $(2,1)$.

True if $F(x,y)$ is conservative.

$P = \sin y$ $\quad \frac{\partial P}{\partial y} = \cos y$ $\checkmark$ $\quad$ both P & Q are continuous on $\mathbb{R}^2$

$Q = x\cos y - \sin y$ $\quad \frac{\partial Q}{\partial x} = \cos y$ $\quad$ so F is conservative

Since F is conservative, $\int_C F \cdot dr = f(\text{endpoint}) - f(\text{start point})$

Solve for f: $\quad \int P\,dx = x\sin(y) + g(y)$

$\int Q\,dy = x\sin(y) + \cos(y) + h(x)$

works if $f = x\sin(y) + \cos(y)$ $\quad \left(\begin{array}{l} g(y) = \cos(y) \\ h(x) = 0 \end{array} \right.$

So $\int_C F \cdot dr = f(2,1) - f(1,0)$

$= (2\sin(1) + \cos(1)) - (1 \cdot 0 + \cos(0))$

$= 2\sin(1) + \cos(1) - 1$