Math 2110Q - Multivariable Calculus
Name:

Section 16.2 Worksheet

1. Evaluate the line integral $\int_C \frac{x}{y} ds$ where $C: x = t^3, y = t^4, 1 \le t \le 2$.

$$x'(t) = 3t^2 \qquad y'(t) = 4t^3$$

$$\int_1^2 \frac{t^3}{t^4}\sqrt{(3t^2)^2 + (4t^3)^2}\, dt = \int_1^2 \frac{1}{t}\sqrt{9t^4 + 16t^6}\, dt$$

$$= \int_1^2 \frac{t^2}{t}\sqrt{9+16t^2}\, dt \qquad u = 9+16t^2 \quad \frac{du}{dt} = 32t \qquad = \frac{1}{32}\frac{2}{3}(9+16t^2)^{3/2}\Big|_1^2$$

$$= \frac{1}{16\cdot 3}\left[(9+64)^{3/2} - (9+16)^{3/2}\right] = \frac{1}{48}\left[(73)^{3/2} - (25)^{3/2}\right]$$

2. Evaluate the line integral $\int_C xy^4 ds$ where C is the right half of the circle $x^2 + y^2 = 16$.

$$x^2+y^2=16 \longrightarrow \begin{array}{l} x = 4\cos t \\ y = 4\sin t \\ -\frac{\pi}{2} \le t \le \frac{\pi}{2} \end{array} \qquad \begin{array}{l} x'(t) = -4\sin t \\ y'(t) = 4\cos t \end{array}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\cos t\,(4\sin t)^4\sqrt{16\sin^2 t + 16\cos^2 t}\, dt = 4^6 \int_{-\frac{\pi}{2}}^{\pi/2} \cos t \sin^4 t\, dt$$

$$= 4^6 \frac{\sin^5 t}{5}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4^6}{5}\cdot 2$$

3. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is given by the vector function $\mathbf{r}(t)$.

(a) $\mathbf{F}(x, y) = xy^2\mathbf{i} - x^2\mathbf{j}$, $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j}$, $0 \le t \le 1$.

$$\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\, dt$$

$$r'(t) = 3t^2 i + 2t j$$

$$x(t) = t^3, \quad y(t) = t^2$$

$$F(r(t)) = F(x(t), y(t)) = t^3 \cdot (t^2)^2 i - (t^3)^2 j$$

$$= \int_0^1 (t^7 i - t^6 j) \cdot (3t^2 i + 2t j)\, dt = \int_0^1 3t^9 + (-2)t^7\, dt$$

$$= \frac{3}{10}t^{10} - \frac{1}{4}t^8 \Big|_0^1 = \frac{3}{10} - \frac{1}{4} = \frac{1}{20}$$

(b) $\mathbf{F}(x, y, z) = \sin x\mathbf{i} + \cos y\mathbf{j} + xz\mathbf{k}$, $\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$.

$$r'(t) = 3t^2 i - 2t j + k$$

$$F(r(t)) = F(x(t), y(t), z(t))$$

$$x(t) = t^3$$

$$y(t) = -t^2$$

$$z(t) = t$$

$$= \sin(t^3) i + \cos(-t^2) j + t^3 \cdot t\, k$$

$$\int_C F \cdot dr = \int_0^1 (\sin(t^3) i + \cos(-t^2) j + t^4 k) \cdot (3t^2 i - 2t j + k)\, dt$$

$$= \int_0^1 3t^2 \sin(t^3) + (-2t)\cos(-t^2) + t^4\, dt$$

$$= -\cos(t^3) + \sin(-t^2) + \frac{t^5}{5} \Big|_0^1 = \left(-\cos(1) + \sin(-1) + \frac{1}{5}\right) - (-1)$$