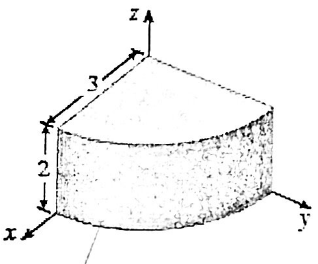
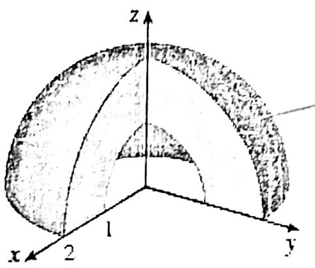
Math 2110Q - Multivariable Calculus
Name:

Section 15.8 Worksheet

1. Set up the triple integral of an arbitrary continuous function $f(x, y, z)$ in cylindrical or spherical coordinates over the solid shown:



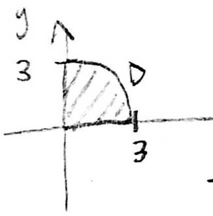


$E = \{ (\rho, \theta, \phi) : 1 \le \rho \le 2, \frac{\pi}{2} \le \theta \le 2\pi, 0 \le \phi \le \frac{\pi}{2} \}$

$\int_0^{\pi/2} \int_{\pi/2}^{2\pi} \int_1^{2} f(\rho \sin\phi\cos\theta, \rho \sin\phi\sin\theta, \rho\cos\phi)\, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

Triple Type I.

$0 \le z \le 2$



$D = \{ (r, \theta) : 0 \le r \le 3, 0 \le \theta \le \pi/2 \}$

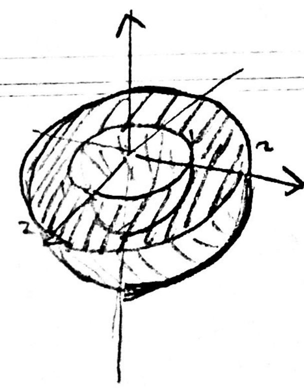
$\int_0^{\pi/2} \int_0^{3} \int_0^{2} f(r\cos\theta, r\sin\theta, z)\, r \, dz \, dr \, d\theta$

2. Sketch the solid whose volume is given by the integral and evaluate the integral: $\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^2 \sin\phi d\rho d\phi d\theta$.

dV

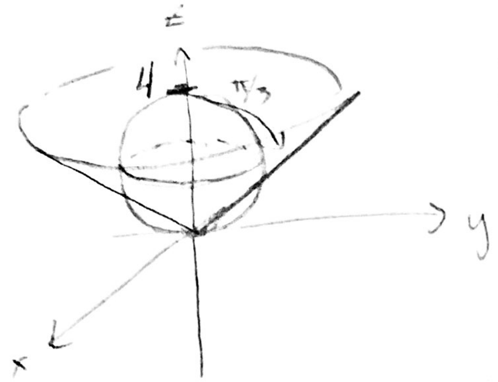
$0 \le \theta \le 2\pi$ $1 \le \rho \le 2$

$\frac{\pi}{2} \le \phi \le \pi$



← upside down cup.

3. Find the volume of the solid that lies above the cone $\phi = \frac{\pi}{3}$ and below the sphere $\rho = 4\cos\phi$.



E = ice cream cone: $\{(\rho, \theta, \phi):$

$0 \le \rho \le 4\cos\phi$

$0 \le \phi \le \pi/3$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^{4\cos\phi} dV = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{4\cos\phi} \rho^2 \cdot \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/3} \frac{\rho^3}{3}\sin\phi \Big|_0^{4\cos\phi} d\phi \, d\theta = \int_0^{2\pi}\int_0^{\pi/3} \frac{64}{3}\cos^3\phi \sin\phi \, d\phi \, d\theta = \frac{64}{3}\int_0^{2\pi} \left[-\frac{\cos^4\phi}{4}\right]_0^{\pi/3} d\theta$$

$$= \frac{64}{12}\int_0^{2\pi} -\left[\left(\tfrac{1}{2}\right)^4 - 1^4\right] d\theta$$

$$= \left(\frac{16}{3}\right)\left(\frac{+15}{16}\right)[2\pi - 0]$$

$$= +5(2\pi) = \boxed{10\pi}$$

4. Set up but do not evaluate $\int\int\int_E y^2 dV$ where E is the solid hemisphere $x^2 + y^2 + z^2 \le 9$ and $y \ge 0$.