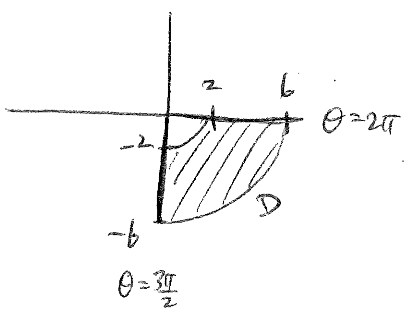
Math 2110Q - Multivariable Calculus
Name:

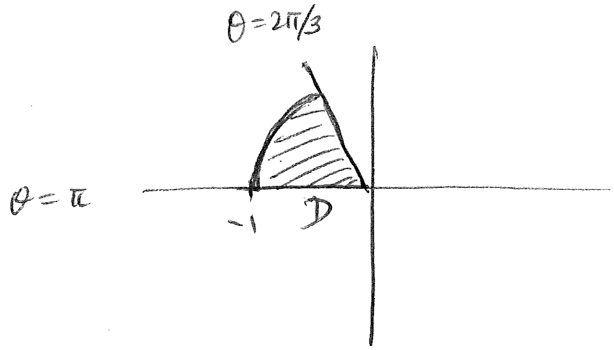
Section 15.3 Worksheet

1. Draw the following regions:

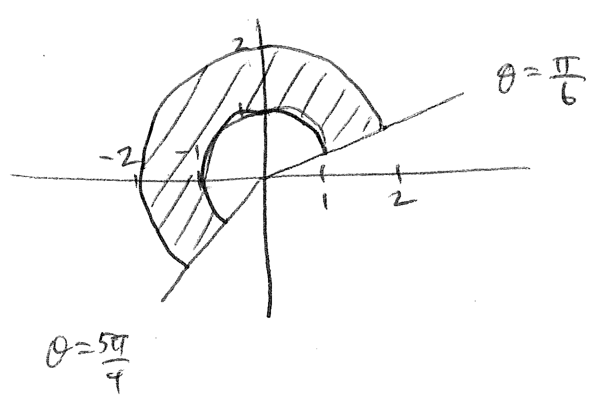
   (a) $D = \left\{(r, \theta) \mid \frac{3\pi}{2} \leq \theta \leq 2\pi, 2 \leq r \leq 6\right\}$



   (b) $D = \left\{(r, \theta) \mid \frac{2\pi}{3} \leq \theta \leq \pi, 0 \leq r \leq 1\right\}$



   (c) $D = \left\{(r, \theta) \mid \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{4}, 1 \leq r \leq 2\right\}$.



2. Evaluate the iterated integral by converting to polar coordinates:

$$\boxed{\int_0^1 \int_y^{\sqrt{2-y^2}}} 3\,(x+y)\,dxdy.$$

$\downarrow$

Type II:

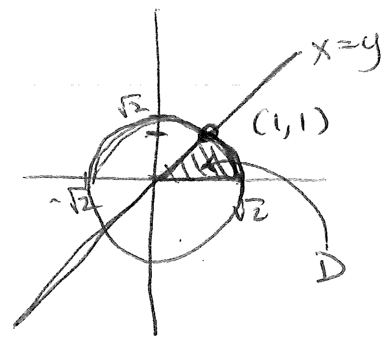
$0 \le y \le 1$

$y \le x \le \sqrt{2-y^2}$

$x = r\cos\theta$

$y = r\sin\theta$

$dxdy = r \cdot drd\theta$



$D = \{(r,\theta) : 0 \le r \le 1,\ 0 \le \theta \le \frac{\pi}{4}\}$

$x \le \sqrt{2-y^2}$

$x^2 \le 2 - y^2$

$x^2 + y^2 \le 2$ circle radius $\sqrt{2}$

$$\int_0^{\pi/4}\int_0^1 3(r\cos\theta + r\sin\theta)\,r\,dr\,d\theta$$

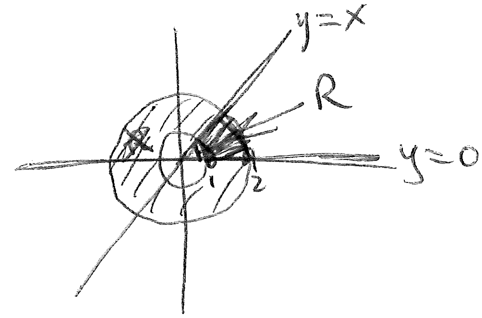
$$= \int_0^{\pi/4}\int_0^1 3r^2(\cos\theta + \sin\theta)\,dr\,d\theta = \int_0^{\pi/4}(\cos\theta + \sin\theta)\,d\theta \int_0^1 3r^2\,dr = \left[\cancel{\sin(\tfrac{\pi}{4}) - \cos(\tfrac{\pi}{4})} - \sin(0) + \cos(0)\right]\left[1-0\right]$$

$$= (+1)(1) = \boxed{1}$$

3. Evaluate $\displaystyle\iint_R \tan^{-1}\left(\frac{y}{x}\right) dA$ where $R = \{(x,y) \mid 1 \le x^2 + y^2 \le 4, 0 \le y \le x\}$.

$R = \{(r,\theta) : 1 \le r^2 \le 4,\ 0 \le \theta \le \pi/4\}$

$(1 \le r \le 2)$



$$\int_0^{\pi/4}\int_1^2 \tan^{-1}\left(\frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}\right) r\,dr\,d\theta$$

$$= \int_0^{\pi/4}\int_1^2 \tan^{-1}(\tan\theta)\,r\,dr\,d\theta$$

$$= \int_0^{\pi/4}\int_1^2 r\,dr\,d\theta = \int_0^{\pi/4} d\theta \cdot \int_1^2 r\,dr = \left(\frac{\pi}{4}\right)\left(\frac{2^2}{2} - \frac{1}{2}\right) = \boxed{\frac{3\pi}{8}}$$