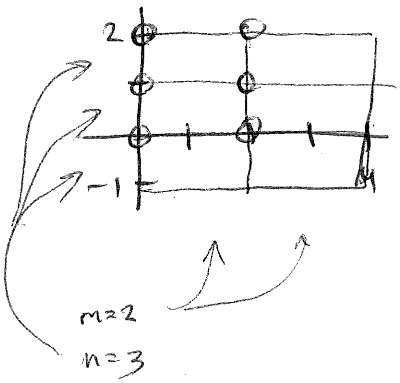
Math 2110Q - Multivariable Calculus
Name:

Section 15.1/15.2 Worksheet

1. Use a Riemann sum to estimate the value of $\iint_R (1 - xy^2)\,dA$ where $R = [0,4]\times[-1,2]$ with $m = 2, n = 3$. Take the sample points to be the upper left corners of the rectangles.



$\Delta A = 2 \times 1 = 2$

$\iint_R (1-xy^2)\,dA \approx 2\big(f(0,0)+f(0,1)+f(0,2) + f(2,0)+f(2,1)+f(2,2)\big)$

where $f = 1 - xy^2$

$= 2(1+1+1+1+(-1)+(-7))$

$= 2(-4) = \boxed{-8}$

2. Calculate the iterated integral

$$\int_{-1}^{2}\int_{0}^{4} (1 - xy^2)\,dx\,dy.$$

$$= \int_{-1}^{2} \left[1 - \frac{x^2y^2}{2}\right]_0^4 dy = \int_{-1}^{2} \left[\left(1 - \frac{16y^2}{2}\right) - (1-0)\right] dy$$

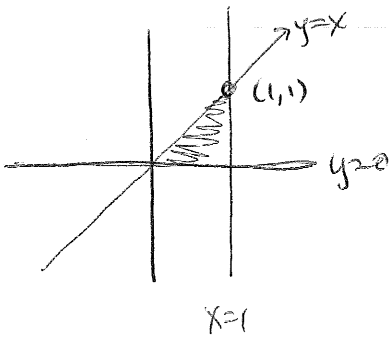
$$= \int_{-1}^{2} -\frac{16y^2}{2}\,dy = -\left[\frac{16y^3}{6}\right]_{-1}^{2} = -\left[\frac{8}{3}(2)^3 - \frac{8}{3}(-1)^3\right]$$

$$= -\left[\frac{64}{3} + \frac{8}{3}\right]$$

$$= -\frac{72}{3} = \boxed{-24}$$

3. Express the following regions D as a region of type I and also as a region of type II. Then evaluate the double integral in two ways.

(a) $\iint_D x\,dA$, where D is bounded by $\boxed{y = x, y = 0, x = 1.}$



Type I:

$0 \le x \le 1$

$0 \le y \le x$

$\rightarrow \int_0^1 \int_0^x x\,dy\,dx = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$

Type II:

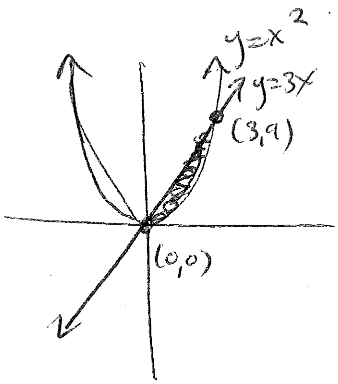
$0 \le y \le 1$

$y \le x \le 1$

$\rightarrow \int_0^1 \int_y^1 x\,dx\,dy = \int_0^1 \frac{1}{2} - \frac{y^2}{2}\,dy = \left[\frac{1}{2}y - \frac{y^3}{6}\right]_0^1$

$= \left[\frac{1}{2} - \frac{1}{6}\right] = \boxed{\frac{1}{3}}$

(b) $\iint_D xy\,dA$, where D is enclosed by the curves $\boxed{y = x^2,\ y = 3x.}$



Intersection point: $x^2 = 3x \rightarrow x(x-3) = 0, \quad x = 0, 3$

Type I:

$0 \le x \le 3$

$x^2 \le y \le 3x$

$\int_0^3 \int_{x^2}^{3x} xy\,dy\,dx = \int_0^3 \frac{xy^2}{2}\Big|_{x^2}^{3x}\,dx$

$= \int_0^3 \frac{9x^3}{2} - \frac{x^5}{2}\,dx = \left[\frac{9}{8}x^4 - \frac{x^6}{12}\right]_0^3$

$= \frac{9}{8}(3)^4 - \frac{(3)^6}{12} = \frac{9 \cdot 81}{8} - \frac{243}{4}$

$= \boxed{\frac{243}{8}}$

$y = x^2 \rightarrow x = \pm\sqrt{y}$

$y = 3x \rightarrow x = \frac{y}{3}$

Type II:

$0 \le y \le 9$

$\frac{y}{3} \le x \le \sqrt{y}$

$\int_0^9 \int_{\frac{y}{3}}^{\sqrt{y}} xy\,dx\,dy$