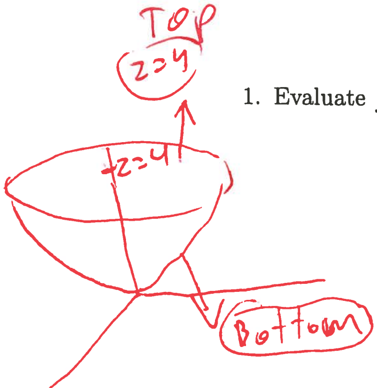
KEY

Math 2110Q - Multivariable Calculus
Name:

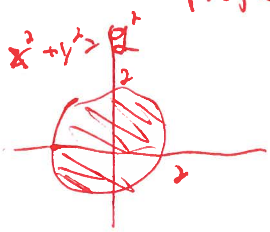
Section 15.8 Worksheet

1. Evaluate $\int\int\int_E z\,dV$ where E is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.



$$\int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} z\, r\,dz\,dr\,d\theta$$

projection $= \int_0^{2\pi}\int_0^{2} \frac{z^2}{2}\, r \Big|_{z=r^2}^{z=4} dr\,d\theta = \int_0^{2\pi}\int_0^{2} \left[8r - \frac{r^5}{2}\right] dr\,d\theta$

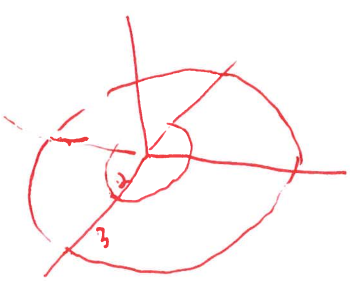


$= \left[\int_0^{2\pi} d\theta\right]\left[\int_0^{2} 8r - \frac{r^5}{2}\, d\theta\right] = 2\pi\left[4r^2 - \frac{r^6}{12}\right]_0^2$

Bottom: $z = x^2+y^2 = \boxed{r^2}$

$= \boxed{\frac{64}{3}\pi}$

2. Set up but do not evaluate $\int\int\int_E x\,dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.



$$\int_0^{2\pi}\int_2^{3}\int_0^{r\cos\theta + r\sin\theta + 5} [r\cos\theta]\, r\,dz\,dr\,d\theta$$

Top function

$z = x + y + 5$

$= r\cos\theta + r\sin\theta + 5$

Bottom function

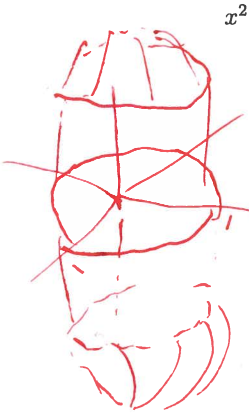
$z = 0$

Remember that

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$$\boxed{r^2 = x^2 + y^2}$$

3. Set up but do not evaluate an expression for the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.



Top function

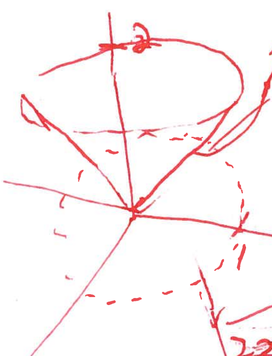
$$z = \sqrt{4 - x^2 - y^2} = \sqrt{4 - r^2}$$

Bottom function

$$z = -\sqrt{4 - r^2}$$

$$\int_0^{2\pi} \int_0^1 \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} 1 \cdot r \, dz \, dr \, d\theta$$

4. Set up but do not evaluate $\int\int\int_E x^2 dV$ where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ and below the cone $z^2 = 4x^2 + 4y^2$.



$\boxed{z = 2}$

$\boxed{z = 0}$ → Bottom

$$z^2 = 4x^2 + 4y^2 = 4(x^2 + y^2)$$

Top $\Rightarrow$ $z = 2\sqrt{x^2 + y^2} = 2\sqrt{r^2} = 2r$

$\Rightarrow \boxed{z = 2r}$

When $x^2 + y^2 = 1$

get

$$z^2 = 4x^2 + 4y^2$$
$$= 4(x^2 + y^2)$$
$$= 4$$

$\Rightarrow z^2 = 4$

$\Rightarrow \boxed{z = 2}$

$$\int_0^{2\pi} \int_0^1 \int_0^{2r} (r\cos\theta)^2 \, r \, dz \, dr \, d\theta$$