Math 2110Q - Multivariable Calculus Section 14.8 Worksheet
Name:

1. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint:

$$f(x,y) = x^2 + y^2, \qquad xy = 1.$$

$\nabla f = \langle 2x, 2y \rangle$ $g(x,y) = xy$, $\nabla g = \langle y, x \rangle$.

$\nabla f = \lambda \nabla g \longrightarrow$ ⓐ $2x = \lambda y$ & ⓒ $xy = 1$.
ⓑ $2y = \lambda x$

ⓐ $\Rightarrow$ $y = \frac{2x}{\lambda}$ plug into ⓑ: $2\left(\frac{2x}{\lambda} = \lambda x\right) \to 4x = \lambda^2 x$

$\to 4x - \lambda^2 x = 0$ $x(4-\lambda^2) = 0$ so $x = 0$ or $4 - \lambda^2 = 0$

If $x=0$, then ⓒ becomes $0 = 1$, impossible.
Thus $4 - \lambda^2 = 0$, $\lambda^2 = 4$, $\lambda = \pm 2$.

If $\lambda = 2$, ⓐ $\Rightarrow 2x = 2y$ so $x = y$. Then plugged into ⓒ get
$x^2 = 1$, so $x = \pm 1$ and so $y = \pm 1$ (signs taken simultaneously).

If $\lambda = -2$, ⓐ $\Rightarrow 2x = -2y$ so $x = -y$. Then plugged into ⓒ get
$-x^2 = 1$, impossible. Thus only points to test are
$(x,y) = (1,1)$ and $(-1,-1)$.

$f(1,1) = 2$, $f(-1,-1) = 2$. abs max or min?
well, another point on $xy=1$ is $(x,y) = (100, \frac{1}{100})$
and $f(100, \frac{1}{100}) \approx 100^2 > 2$, so 2 must be the absolute min.

2. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint:

$$f(x, y, z) = 2x + 2y + z, \qquad x^2 + y^2 + z^2 = 9.$$

$\nabla f = \langle 2, 2, 1 \rangle$

$g(x,y,z) = x^2+y^2+z^2$

$\nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g \rightarrow$ ⓐ $2 = \lambda 2x$ & ⓓ $x^2+y^2+z^2=9$

ⓑ $2 = \lambda 2y$

ⓒ $1 = \lambda 2z$

ⓐ, ⓑ, ⓒ became: $\frac{1}{\lambda} = x$, $\frac{1}{\lambda} = y$, $\frac{1}{2\lambda} = z$

sub into ⓓ: $\left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 9$

$$\frac{1}{\lambda^2} + \frac{1}{\lambda^2} + \frac{1}{4\lambda^2} = 9$$

$$\frac{4}{4\lambda^2} + \frac{4}{4\lambda^2} + \frac{1}{4\lambda^2} = 9$$

$$\frac{9}{4\lambda^2} = 9$$

$$4\lambda^2 = 1$$

$$\lambda^2 = \frac{1}{4},$$

$$\lambda = \pm \frac{1}{2}.$$

If $\lambda = \frac{1}{2}$, $x = 2$, $y = 2$, $z = 1$ $\qquad f(2,2,1) = 9$ ← abs max

If $\lambda = -\frac{1}{2}$, $x = -2$, $y = -2$, $z = -1$ $\qquad f(-2,-2,-1) = -9$ ← abs. min.