Math 2110Q - Multivariable Calculus
Name:

Section 14.7 Worksheet

1. Find the local maximum and minimum values and saddle points for

$$f(x,y) = xy - 2x - 2y - x^2 - y^2.$$

$f_x = y - 2 - 2x$ $\quad f_{xx} = -2 \quad f_{xy} = f_{yx} = 1$

$f_y = x - 2 - 2y$ $\quad f_{yy} = -2$

critical points: $\longrightarrow (-2,-2)$

$f_x = 0, \ f_y = 0$ $\quad y = 2 + 2(-2) = -2$

$0 = y - 2 - 2x$ $\quad 0 = x - 2 - 2y$

$2 + 2x = y$ $\quad 0 = x - 2 - 2(2+2x)$

$0 = x - 2 - 4 - 4x$

$0 = -3x - 6 \longrightarrow x = -2$

so $D(-2,-2) = (-2)(-2) - 1^2$
$= 3 > 0$

and $f_{xx}(-2,-2) = -2 < 0$

so by 2nd deriv. test,

$(-2,-2)$ is an local max.

2. Find three positive number whose sum is 100 and whose product is maximum.

$x + y + z = 100$ $\quad P = xyz$

$\longrightarrow z = 100 - x - y \longrightarrow P = xy(100 - x - y) = 100xy - x^2y - xy^2$

so $P_x = 100y - 2xy - y^2$ $\quad P_y = 100x - x^2 - 2xy$

crit pts: $P_x = 0 \quad (100 - 2x - y)y = 0 \longrightarrow$ Two cases: (b) $100 - 2x - y = 0$ or (a) $y = 0$

$P_y = 0 \quad (100 - 2y - x)x = 0$

(a) If $y = 0$, 2nd equation becomes $(100 - x)x = 0$, so $x = 100$ or $x = 0$.

crit points: $(100, 0), (0, 0)$

(b) If $100 - 2x - y = 0$, from 2nd equation we have either (I) $100 - 2y - x = 0$ or (II) $x = 0$

(II) $100 - 2x - y = 0$ and $x = 0$, so $100 - y = 0$ so $y = 100$

crit point $(0, 100)$

(I) $y = 100 - 2x$, sub into (I): $100 - 2(100 - 2x) - x = 0 \rightarrow 100 - 200 + 4x - x = 0$

$3x = 100, \quad x = \frac{100}{3}$

so $y = \frac{100}{3}$

crit point: $\left(\frac{100}{3}, \frac{100}{3}\right)$.

Clearly, the points $(x,y) = (0,0), (100,0)$ and $(0,100)$ give a product of 0.

we compute $P_{xx} = -2y$, $P_{yy} = -2x$, $P_{xy} = 100 - 2x - 2y$, $D = 4xy - (100 - 2x - 2y)^2$

$D\left(\frac{100}{3}, \frac{100}{3}\right) > 0$ and $P_{xx}\left(\frac{100}{3}, \frac{100}{3}\right) < 0$, so $\left(\frac{100}{3}, \frac{100}{3}\right)$ is a max.

3. Find the dimensions of the box with volume 1000 cm^3 that has minimal surface area.