Math 2110Q - Multivariable Calculus
Name:

Section 14.6 Worksheet

1. Find the directional derivative of $f(x,y) = e^x \sin y$ at $(0, \frac{\pi}{3})$ in the direction of $\mathbf{v} = \langle -6, 8 \rangle$.

$\nabla f = \langle f_x, f_y \rangle$ so $\nabla f = \langle e^x \sin y, e^x \cos y \rangle$

$\bar{v} = \langle -6, 8 \rangle$ so $\bar{u} = \frac{\bar{v}}{|\bar{v}|} = \langle -6/10, 8/10 \rangle = \langle \frac{-3}{5}, \frac{4}{5} \rangle$

so $D_{\bar{u}} f(0, \frac{\pi}{3}) = \nabla f(0, \frac{\pi}{3}) \cdot \langle \frac{-3}{5}, \frac{4}{5} \rangle$

$= \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle \cdot \langle \frac{-3}{5}, \frac{4}{5} \rangle$

$= \frac{-3\sqrt{3}}{10} + \frac{4}{10} = \frac{4 - 3\sqrt{3}}{10}$

2. Suppose that the temperature at a point (x,y,z) in space is given by $T(x,y,z) = 80/(1 - x^2 - 2y^2 - 3z^2)$ where T is measured in degrees Celsius and x, y, z in meters. In which direction does the temperature increase fastest at the point $(1, 1, -2)$? What is the maximum rate of increase?

direction of fastest increase: $\nabla f(1,1,-2)$

$$\nabla f = \left\langle \frac{-80}{(1+x^2+2y^2+3z^2)^2} \cdot (2x), \frac{-80}{(1+x^2+2y^2+3z^2)^2} \cdot (4y), \frac{-80}{(1+x+2y^2+3z^2)^2} (6z) \right\rangle$$

$$\text{so } \nabla f(1,1,-2) = \left\langle \frac{-160(1)}{(1+1+2+12)^2}, \frac{-320(1)}{(1+1+2+12)^2}, \frac{-480(-2)}{(1+1+2+12)^2} \right\rangle$$

$$\nabla f(1,1,-2) = \left\langle \frac{-5}{8}, \frac{-5}{4}, \frac{15}{4} \right\rangle$$

maximum rate of increase: $|\nabla f(1,1,-2)|$

$$\left| \left\langle \frac{-5}{8}, \frac{-5}{4}, \frac{15}{4} \right\rangle \right| = \frac{5\sqrt{41}}{8}$$

3. Find all points in which the direction of fastest change of the function $f(x,y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

$\langle 1,1 \rangle$

∇f

$\nabla f = \langle 2x-2, 2y-4 \rangle$

as long as the gradient is a multiple of $\langle 1,1 \rangle$, the direction of fastest change will be $\langle 1,1 \rangle$.

$\nabla f = \lambda \cdot \langle 1,1 \rangle$

$2x-2 = \lambda$

$2y-4 = \lambda$

$x = \frac{\lambda}{2} + 1$

$y = \frac{\lambda}{2} + 2$

so all points of the form

$\left(\frac{\lambda}{2}+1, \frac{\lambda}{2}+2\right)$

for some λ