1. Find an equation of the tangent plane to the surface $z = \frac{x}{y^2}$ at the point $(-4, 2, 1)$.

$f(x,y) = \frac{x}{y^2}$

$z = z_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$\frac{\partial f}{\partial x} = \frac{1}{y^2}$ $\quad \frac{\partial f}{\partial y} = \frac{-2x}{y^3}$

$z = 1 + \frac{1}{4}(x+4) + 1\cdot(y-2)$

$\frac{\partial f}{\partial x}(-4,2) = \frac{1}{4}$ $\quad \frac{\partial f}{\partial y}(-4,2) = \frac{8}{8} = 1$

$z = 1 + \frac{x}{4} + 1 + y - 2$

$\boxed{z = \frac{x}{4} + y}$

2. Explain why the function is differentiable at the given point. Then find the linearization $L(x, y)$ of the function at that point.

$$f(x, y) = 1 + x\ln(xy - 5), \quad (2, 3)$$

$f_x = \frac{xy}{xy-5} + \ln(xy-5)$

$f_y = \frac{x^2}{xy-5}$

both f_x and f_y exist at $(2,3)$ and are continuous at $(2,3)$. Thus f is differentiable at $(2,3)$.

$L(x,y) = z_0 + f_x(2,3)(x-2) + f_y(2,3)(y-3)$

$L(x,y) = f(2,3) + 6(x-2) + 4(y-3)$

$L(x,y) = 1 + 6x - 12 + 4y - 12$

$L(x,y) = 6x + 4y - 23$

3. Use the chain rule to find $\frac{dz}{dt}$ for $z = \frac{x-y}{x+2y}$

~~z = frac x − yx + 2y~~; $x = t^2$, $y = 2t - 1$.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} \qquad \frac{dx}{dt} = 2t \qquad \text{so } \frac{dz}{dt} = \left(\frac{3y}{(x+2y)^2}\right)(2t) + \left(\frac{-2x-y}{(x+2y)^2}\right)(2)$$

$$\frac{\partial z}{\partial x} = \frac{(x+2y)(1) - (x-y)(1)}{(x+2y)^2} = \frac{3y}{(x+2y)^2} \qquad \frac{dy}{dt} = 2$$

$$\frac{\partial z}{\partial y} = \frac{(x+2y)(-1) - (x-y)(1)}{(x+2y)^2} = \frac{-2x-y}{(x+2y)^2}$$

4. Find $\frac{dz}{dt}$ of the above function by first substituting for x and y in the equation of z.

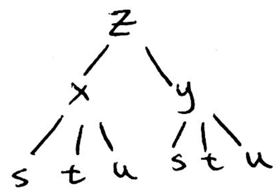
$$z = \frac{t^2 - (2t-1)}{t^2 + 2(2t-1)} = \qquad \frac{dz}{dt} = \frac{(t^2+4t-2)(2t-2) - (t^2-2t+1)(2t+4)}{(t^2+4t-2)^2}$$

$$z = \frac{t^2-2t+1}{t^2+4t-2}$$

5. Use the chain rule to compute $\frac{\partial z}{\partial u}$ where

$$z = x^4 + x^2 y, \quad x = s + 2t - u, \quad y = stu^2$$

when $s = 4$, $t = 2$, $u = 1$.



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \qquad \text{so} \quad \frac{\partial z}{\partial u} = (4x^3+2xy)(-1) + x^2(2stu)$$

$$\frac{\partial z}{\partial x} = 4x^3 + 2xy \qquad \frac{\partial x}{\partial u} = -1$$

$$\frac{\partial z}{\partial y} = x^2 \qquad \frac{\partial y}{\partial u} = 2stu$$