Math 2110Q - Multivariable Calculus
Name:

Section 14.3 Worksheet

1. Find the first partials of the following
   - $z = (2x+3y)^{10}$
   - $f(x,y) = \frac{x}{(x+y)^2}$

$\frac{\partial z}{\partial x} = 10(2x+3y)^9 \cdot 2$

$\frac{\partial z}{\partial y} = 10(2x+3y)^9 \cdot 3$

$\frac{\partial f}{\partial x} = (x+y)^2 - x(2(x+y))$

$\frac{\partial f}{\partial y} = x(-2(x+y)^{-3})$

2. Find $f_x(2,3)$ for $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$.

$f_x(x,y) = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot (-yx^{-2})$

$f_x(2,3) = \frac{1}{1+\left(\frac{3}{2}\right)^2} \cdot \left(\frac{-3}{4}\right)$

3. Find $f_x(2,1,-1)$ for $f(x,y,z) = \frac{y}{x+y+z}$. $= y(x+y+z)^{-1}$

$f_x = -y(x+y+z)^{-2}$

$f_x(2,1,-1) = -(1)(2+1-1)^{-2} = \frac{-1}{4}$

4. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $x^2 + 2y^2 + 3z^2 = 1$.

$\boxed{\frac{\partial z}{\partial x}}$ $\quad 2x + 0 + 6z\frac{\partial z}{\partial x} = 0 \quad$ so $\quad \frac{\partial z}{\partial x} = \frac{-2x}{6z} = \frac{-x}{3z}$

$\boxed{\frac{\partial z}{\partial y}}$ $\quad 0 + 4y + 6z\frac{\partial z}{\partial y} = 0 \quad$ so $\quad \frac{\partial z}{\partial y} = \frac{-4y}{6z} = \frac{-2y}{3z}$

5. Find all second partials of $f(x, y) = x^3y^5 + 2x^4y$.

$f_x = 3x^2y^5 + 8x^3y \qquad f_y = 5x^3y^4 + 2x^4$

$f_{xx} = 6xy^5 + 24x^2y \qquad f_{xy} = 15xy^4 + 8x^3 \qquad f_{yy} = 20x^3y^3$