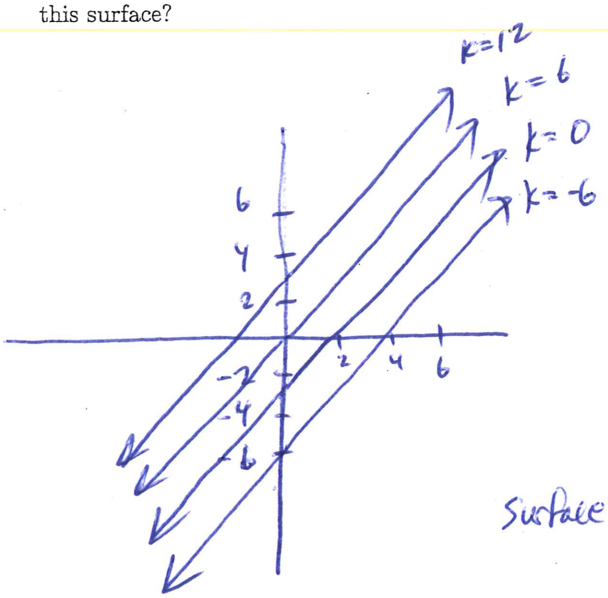
Math 2110Q - Multivariable Calculus

Name:

Section 14.1-14.2 Worksheet

1. Sketch the level curves of $f(x, y) = 6 - 3x - 2y$ for the values $k = -6, 0, 6, 12$. What is the shape of this surface?



$-6 = 6 - 3x - 2y \Rightarrow y = \frac{3x-12}{2}$

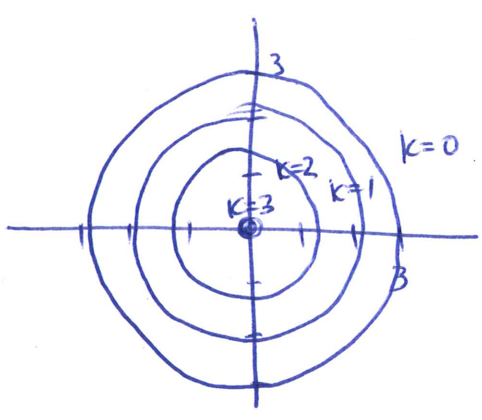
$0 = 6 - 3x - 2y \Rightarrow y = \frac{3x-6}{2}$

$6 = 6 - 3x - 2y \Rightarrow y = \frac{3x}{2}$

$12 = 6 - 3x - 2y \Rightarrow y = \frac{3x+6}{2}$

surface is plane

2. Sketch the level curves of the function $g(x, y) = \sqrt{9 - x^2 - y^2}$ for the values $k = 0, 1, 2, 3$. What is the shape of this surface?



$0 = \sqrt{9-x^2-y^2} \Rightarrow x^2+y^2 = 9$

$1 = \sqrt{9-x^2-y^2} \Rightarrow x^2+y^2 = 8$

$2 = \sqrt{9-x^2-y^2} \Rightarrow x^2+y^2 = 5$

$3 = \sqrt{9-x^2-y^2} = x^2+y^2 = 0$

(top) ~~semi~~ hemisphere

3. Consider the function $f(x, y) = \dfrac{xy^2}{x^2 + y^4}$. We will investigate

$$\lim_{(x,y)\to(0,0)} f(x, y)$$

(a) First show that approaching $(0, 0)$ along the x-axis and y-axis give you the same limit.

$y=0$ $\quad$ $x=0$

x-axis: $\lim f(x,y) = \lim \frac{0}{x^2} = 0$

y-axis: $\lim f(x,y) = \lim \frac{0}{y^4} = 0$

(b) Show that approaching $(0, 0)$along lines of the form $y = kx$ still gives the same limit.

$$\lim f(x,y) = \lim \frac{x(k^2x^2)}{x^2 + k^4x^4} = \lim \frac{k^2x^3}{x^2(1+k^4x^2)} = \lim \frac{k^2x}{1+k^4x^2} = \frac{0}{1} = 0$$

(c) Finally, approach the point $(0, 0)$ along the parabola $x = y^2$. What is the limit along this path?

$$\lim f(x,y) = \lim \frac{y^4}{y^4+y^4} = \lim \frac{y^4}{2y^4} = \lim \frac{1}{2} = \frac{1}{2}$$

(d) Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist or not?

No, b/c different directions yield different limits.