Math 2110Q - Multivariable Calculus
Name:

Section 13.2-13.3 Worksheet

1. Find the derivative of $\mathbf{r}(t) = \frac{1}{1+t}\mathbf{i} + \frac{t}{1+t}\mathbf{j} + \frac{t^2}{1+t}\mathbf{k}$.

$$r'(t) = \frac{-1}{(1+t)^2}\,i + \frac{1}{(1+t)^2}\,j + \frac{t(t+2)}{(1+t)^2}\,k$$

2. Find the parametric equation for the tangent line to the curve with the given parametric equation at the specified point:

$$x = e^t,\ y = te^t\ z = te^{t^2}\ (1, 0, 0).$$

$$r(t) = \langle e^t, te^t, te^{t^2} \rangle$$

$$r'(t) = \langle e^t, (1+t)e^t, (1+2t^2)e^{t^2} \rangle$$

when does the curve pass through $(1,0,0)$?

$r(t) = (1,0,0)$

$$\left.\begin{array}{l} e^t = 1 \\ te^t = 0 \\ te^{t^2} = 0 \end{array}\right\} t = 0$$

tangent vector at that time? $r'(0) = \langle 1, 1, 1 \rangle$

equation of line: $\bar{l}(t) = \langle 1,0,0 \rangle + t\langle 1,1,1 \rangle$.

3. Find the length of the curve $\mathbf{r}(t) = 12t\mathbf{i} + 8t^{3/2}\mathbf{j} + 3t^2\mathbf{k}$ for $0 \le t \le 1$.

$$r(t) = \langle 12t,\ 8t^{3/2},\ 3t^2 \rangle$$

$$r'(t) = \langle 12,\ 12t^{1/2},\ 6t \rangle$$

$$\int_0^1 |r'(t)|\,dt = \int_0^1 \sqrt{144 + 144t + 36t^2}\,dt = \int_0^1 6\sqrt{4 + 4t + t^2}\,dt$$

$$= 6\int_0^1 \sqrt{(2+t)^2}\,dt = 6\int_0^1 (2+t)\,dt = 6\left[2t + \frac{t^2}{2}\right]_0^1$$

$$= 6\left[\frac{5}{2}\right] - 6[0] = \boxed{15}$$

4. Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point $(6, 18, 36)$.
(Hint: Let $x = t$ to find the equation of the curve.)

Equation of curve: $x = t \longrightarrow t^2 = 2y,\ y = \frac{t^2}{2}$

$$y = \frac{t^2}{2} \longrightarrow 3z = \frac{t(t)^2}{2}$$

$$z = \frac{t^3}{6}$$

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$$\bar{r}(t) = \left\langle t,\ \frac{t^2}{2},\ \frac{t^3}{6} \right\rangle \qquad \bar{r}(t) = \langle 6, 18, 36 \rangle \Rightarrow t = 6$$

$$r'(t) = \left\langle 1,\ t,\ \frac{t^2}{2} \right\rangle$$

$$\int_0^6 |r'(t)|\,dt = \int_0^6 \sqrt{1 + t^2 + \frac{t^4}{4}}\,dt$$

$$= \int_0^6 \frac{1}{2}\sqrt{4 + 4t^2 + t^4}\,dt = \frac{1}{2}\int_0^6 \sqrt{(2+t^2)^2}\,dt = \frac{1}{2}\int_0^6 2 + t^2\,dt$$

$$= \frac{1}{2}\left[2t + \frac{t^3}{3}\right]_0^6 = \frac{1}{2}[12 + 72] = \boxed{42}$$