Math 2110Q - Multivariable Calculus
Name:

Section 12.6-13.1 Worksheet

1. Use cross sections of the form $z = k$, $x = 0$, and $y = 0$ to show that $x^2 + y^2 = z^2$ is a cone.

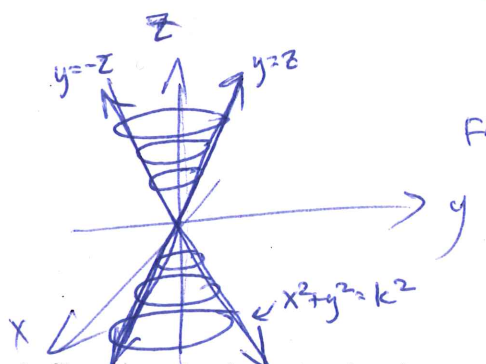
$\boxed{z=k}$: $x^2+y^2=k^2 \rightarrow$ circles of radius k

y-z plane $\boxed{x=0}$: $y^2=z^2 \rightarrow y=z$ or $y=-z$

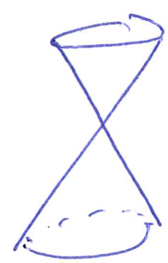
two lines

xz plane $\boxed{y=0}$: $x^2=z^2 \rightarrow x=z$ or $x=-z$

two lines



Final view:



2. Show through substitution that the curve with parametric equations $x = t\cos t, y = t\sin t, z = t$ lies on the above cone. Try to sketch the curve.

sub in:

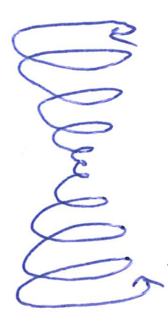
$$(t\cos t)^2 + (t\sin t)^2 = t^2$$

$$t^2(\cos t)^2 + t^2(\sin t)^2 = t^2$$

$$t^2((\cos t)^2 + (\sin t)^2) = t^2$$

$$t^2(1) \overset{\checkmark}{=} t^2$$

Curve looks like:



3. Find the limit

$$\lim_{t \to 0} \left(e^{-t} i + \frac{\sin t}{t} j + (-1 + 3t^2) k \right).$$

$= e^0 i + 1 j + (-1) k = i + j - k$

4. At what points (there are two) does the curve $\mathbf{r}(t) = ti + (6t - t^2)k$ intersect the paraboloid $z = x^2 + y^2$?

$r(t) = \langle t, 0, 6t - t^2 \rangle$

want to satisfy $z = x^2 + y^2$

$$\left\{ \begin{array}{l} x = t \\ y = 0 \\ z = 6t - t^2 \end{array} \right.$$

$6t - t^2 = (t)^2 + (0)^2$

$6t - 2t^2 = 0$

$2t(3 - t) = 0$

$t = 0, \quad t = 3$

$r(0) = \langle 0, 0, 0 \rangle$

$r(3) = \langle 3, 0, 9 \rangle$