Math 2110Q - Multivariable Calculus **Section 12.5 Worksheet**
Name:

1. Find the equation of a line parallel to

$$\frac{x-5}{2} = 2(y-3) = \frac{z-15234}{3}$$

passing through the point $P_0(2,1,-2)$.

For a line, need a point and direction vector.

Point: $P_0(2,1,-2)$

Direction: parallel to $\frac{x-5}{2} = 2(y-3) = \frac{z-15234}{3}$ which has direction vector $\langle 2, \frac{1}{2}, 3\rangle$. So has same direction.

Equation: $\boxed{\frac{x-2}{2} = \frac{y-1}{\frac{1}{2}} = \frac{z+2}{3}}$

2. Find an equation of the plane through the origin and the points $(3,-3,8)$ and $(8,1,2)$.

O P Q

$\overline{OP} = \langle 3,-3,8\rangle$

$\overline{OQ} = \langle 8,1,2\rangle$

normal vector to plane: $\overline{OP} \times \overline{OQ}$

$= \langle -14, 58, 27\rangle$

point on plane: $O = \langle 0,0,0\rangle$ so eqn: $\boxed{-14x + 58y + 27z = 0}$

3. Find the equation of the line given by the intersection of the two planes:

$$x + y + z = 1, \quad x - 2y + 3z = 1.$$

Hint: A vector is in a plane only if it is orthogonal to the normal vector of the plane.

$\vec{n}_1 = \langle 1,1,1\rangle$

$\vec{n}_2 = \langle 1,-2,3\rangle$

direction of line is orthogonal to $\vec{n}_1$ and $\vec{n}_2$

so: direction vector $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle 5,-2,-3\rangle$

point on line: solve system of equations:

$$\begin{cases} x+y+z=1 \\ x-2y+3z=1 \end{cases}$$

subtracting yields: $3y - 2z = 0$

$y = \frac{2}{3}z$

pick $z=0$, then $y=0$, $x=1$. $P_0 = \langle 1,0,0\rangle$

$\boxed{\vec{r}(t) = \langle 1,0,0\rangle + t\langle 5,-2,-3\rangle}$

4. Find an equation of the plane that passes through the point $(1, 3, 4)$ and contains the line

$$x = 4t,\ y = 1 + t,\ z = 3 - t.$$

Hint: Find three points on the plane and then proceed as in #~~1~~2

Alternative hint: The direction of the line gives you a vector in the plane.

$t=0 \rightarrow \langle 0, 1, 3 \rangle$ on line (on plane)

$t=1 \rightarrow \langle 4, 2, 2 \rangle$ on line (on plane).

Now do same thing as in #2 with these points.

$$-3(x-0)+5(y-1)-7(z-3)=0$$

5. Sketch the plane $2x + 5y + z = 10$.

$y=0, z=0 \Rightarrow x=5$

$x=0, z=0 \Rightarrow y=2$

$x=0, y=0 \Rightarrow z=10$