Math 2110Q - Multivariable Calculus
Name:

Section 12.4 Worksheet

1. Let $\vec{a} = \langle 1, 3, -1\rangle$ and $\vec{b} = \langle 0, 4, -2\rangle$. Compute $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$.

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 0 & 4 & -2 \end{vmatrix} = \begin{array}{l} -6i + 0j + 4k \\ +4i + 2j - 0k \\ \hline -2i + 2j + 4k \end{array}$$

$$\bar{b}\times\bar{a} = -\bar{a}\times\bar{b} = 2i - 2j - 4k$$

2. With the same vectors as above, show that $\vec{a}\times\vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$ by computing dot products.

$$(\bar{a}\times\bar{b})\cdot\bar{a} = \langle -2, 2, 4\rangle \cdot \langle 1, 3, -1\rangle = -2 + 6 - 4 = 0$$

$$(\bar{a}\times\bar{b})\cdot\bar{b} = \langle -2, 2, 4\rangle \cdot \langle 0, 4, -2\rangle = 0 + 8 - 8 = 0$$

3. Show this in more generality. Show $(\vec{u}\times\vec{v})\cdot\vec{u} = 0$ for $\vec{u} = \langle u_1, u_2, u_3\rangle, \vec{v} = \langle u_1, u_2, u_3\rangle$.

see book.

4. (a) Compute $k \times (2i - j)$ by converting both vectors to component form first.

$k = \langle 0, 0, 1 \rangle$ $\qquad$ $2i - j = \langle 2, -1, 0 \rangle$

$$k \times (2i - j) = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \begin{array}{l} 0i + 2j + 0k \\ +i - 0j - 0k \\ \hline i + 2j \end{array}$$

$\langle 1, 2, 0 \rangle$

(b) Compute $2(k \times i) - k \times j$ by using your knowledge about the right-hand rule.

$k \times i = j$

$k \times j = -i$

so $\quad 2(k \times i) - k \times j = 2j - (-i) = i + 2j$

(c) What do you know notice about the answers to part (a) and (b)? Can you draw some conclusions about the cross product?

They are the same. The cross product is distributive!