Name:

Test 1 - Practice Questions

1. Which of the following matrices are in row echelon form? Which are in reduced row echelon form?

$$\begin{bmatrix}1&3&5\\2&3&0\\1&0&0\end{bmatrix},\ \begin{bmatrix}1&0&0&0\\0&1&2&0\\0&0&0&1\end{bmatrix},\ \begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix},\ \begin{bmatrix}1&0&2&3\\0&1&0&1\\0&1&2&0\end{bmatrix},\ \begin{bmatrix}1&1&0&1\\0&0&1&0\\0&0&0&1\end{bmatrix},\ \begin{bmatrix}0&0&0&0\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}$$

2. Solve the following system of equations:

$$\begin{array}{rcrcrcr} & & x_2 & + & 5x_3 & = & -4\\ x_1 & + & 4x_2 & + & 3x_3 & = & -2\\ 2x_1 & + & 7x_2 & + & x_3 & = & -2\end{array}$$

3. Solve the following system of equations:

$$\begin{array}{rcrcrcr} 2x_1 & & & - & 6x_3 & = & -8\\ & & x_2 & + & 2x_3 & = & 3\\ 3x_1 & + & 6x_2 & - & 2x_3 & = & -4\end{array}$$

4. (a) Is $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$ in span$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \right\}$? What about $\begin{bmatrix} \pi \\ \log_2 3 \\ 17 \end{bmatrix}$?

   (b) Is $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$? Is $\begin{bmatrix} \pi \\ \log_2 3 \\ 17 \end{bmatrix}$?

5. Let

$$A = \begin{bmatrix} 1 & 4 & -1 \\ 1 & 5 & 0 \\ 0 & 3 & 3 \end{bmatrix}.$$

   (a) Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the span of the columns of A? What about $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$?

   (b) Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ a linear combination of the columns of A? What about $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$?

6. Suppose $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

   (a) Give an example of a vector in span S but not in S.

   (b) Give an example of a vector **NOT** in span S.

7. Find a vector $\vec{x}$ such that

$$\begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 6 & 2 & 4 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

8. Calculate the following matrix products if they are defined, otherwise state they are undefined.

(a) $\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 1 \\ 0 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 0 \\ 4 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & 5 \\ 7 & 13 & 4 \\ -2 & 15 & -17 \end{bmatrix}$

9. (a) Write $\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$?

(b) Is te set $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ linearly independent?

(c) Do these vectors span $\mathbb{R}^3$?

10. Determine whether the following sets are linearly independent:

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

(f) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

11. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the transformation defined by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x + z \\ y + z \end{bmatrix}$$

(a) Show that T is a linear transformation.

(b) Determine the standard matrix for T.

(c) Is T onto?

(d) Is T one-to-one?

12. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}$$

(a) Show that T is a linear transformation.

(b) Determine the standard matrix for T.

(c) Is T onto?

(d) Is T one-to-one?

13. Compute the determinant and calculate the inverses of the following matrices:

(a)
$$\begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 7 & 3 \\ -6 & -3 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 2 & -4 \\ 4 & -6 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

(e)
$$\begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{bmatrix}$$

(f)
$$\begin{bmatrix} 1 & 2 & 0 \\ -2 & 4 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

(g)
$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ -1 & 2 & 0 & -2 \end{bmatrix}$$

14. Compute the inverse of the following matrix:
$$\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$$

15. Use the above inverse to solve the following systems of equations

$$\begin{array}{rcrcr} 2x_1 & + & 3x_2 & = & -6 \\ 2x_1 & + & 2x_2 & = & 15 \end{array}$$

16. Compute the determinant of the following matrices (you may use any method you see fit).

(a) $\begin{bmatrix} 5 & \pi \\ -3 & e \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$

17. Compute the determinant of the following matrices using row/column expansion.

(a) $\begin{bmatrix} 2 & -5 & 4 \\ 0 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ -6 & 0 & 2 & 0 \\ 1 & -4 & 0 & 6 \end{bmatrix}$

18. Complete the following sentences:
    (a) A square matrix A is invertible if and only if $\det A$ ______
    (b) If a square matrix A is singular, the columns of A are linearly ________________
    (c) If a square matrix A is invertible, the matrix equation $A\overline{x} = \overline{b}$ has ________________ solution(s).

19. Suppose A is a square matrix such that $A^4 = 0$. Explain why A cannot be invertible.

20. Suppose that A is a square matrix such that $A^4 = I_n$. Explain why A is invertible.

21. Let A be a 3x3 matrix with $\det A = 4$. What is the determinant of
    (a) A^2
    (b) $3A$